

Electrothermal transport of magnetic (SWCNT) suspensions in microchannels and cilia for solar energy

Ali Khalefa Hagi, Liqaa Zeki Hummady

Department of Mathematics, College of Science, University of Baghdad, Baghdad, Iraq

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ABSTRACT

This study aims to evaluate the electrothermal and hydrodynamic transport properties of an electrically conductive nanofluid containing single-walled carbon nanotubes (SWCNTs) within a microchannel with wavy walls, with the goal of enhancing thermal performance efficiency in solar energy systems. The developed mathematical model is based on formulating the governing equations of motion, momentum, energy, and concentration, under the assumptions of long wavelength and low Reynolds number consistent with the nature of laminar microfluidic flows. The methodology included the use of perturbation techniques to derive approximate analytical solutions for the nonlinear energy equation and to derive precise mathematical formulations for velocity distribution, pressure gradient, pressure increase, flow function, as well as temperature and concentration fields. Numerical and graphical analyses were conducted using the MATHEMATICA software to verify the impact of governing physical parameters on fluid response and heat transfer, with outputs represented through two-dimensional and three-dimensional plots. The results revealed a significant improvement in heat transfer rates when integrating SWCNT nanoparticles, along with enhanced mixing and fluid rotation mechanisms due to the ciliary's peristaltic motion, with a critical impact of physical variables on the phenomenon of entrapment and pressure and temperature distributions. These results demonstrate the suitability of SWCNT-based nanofluids as highly efficient operating media in micro heat exchangers, providing valuable theoretical insights for the development of sustainable heat management and renewable energy technologies.

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Corresponding Author:

Ali Khalefa Hagi

Department of Mathematics, College of Science, University of Baghdad

Baghdad 10071, Iraq

Email: ali.ali2303@sc.uobaghdad.edu.iq

1. INTRODUCTION

Solar energy research is focused on the improvement of photovoltaic-thermal (PV/T) systems to efficiently convert solar energy into electricity and heat. The major hurdle in the development and functioning of such systems is the low thermal conductivity and poor dissipation of the working fluids used in conventional solar-thermal collectors [1]. A major area of concern is that the heat accumulates in the solar collectors and PV modules, which in turn reduces the electrical efficiency, hastens degradation of materials, and limits overall energy conversion capability of these systems. Thus, significant research is currently aimed at designing and developing advanced heat transfer fluids for all types of renewable energy systems to enhance thermal management [2].

Nanofluids are presently considered as a replacement for the conventional heat transfer fluids due to better thermophysical properties. Among the variously studied nanoparticles, single-walled carbon nanotubes (SWCNTs) are considered most due to their exceptionally high thermal conductivity, large surface area, and good electrical properties [3]. Base-fluid suspensions with SWCNTs have been reported to enhance the heat transfer performance applied to solar thermal energy. Apart from that, SWCNT-based materials are very good for PV technologies due to their unique electronic structure, tunable optical properties, and good charge transport and conversion of energy processes [4], [5].

Apart from nanofluid enhancement, studies on microfluidic transport mechanisms have been developed recently for the betterment of thermal management. Cilia-driven transport is an efficient biomimetic technique for fluid mixing to enhance convective heat transfer and control fluid motion at the microscale [6]. Other than electrically conductive nanofluids, external electric and magnetic fields are additional means for flow control in microchannels. The interaction between electromagnetic force and thermal gradients on the transport of nanofluids play a vital role in the hydrodynamic and thermal performance of energy systems [7], [8].

Several studies have been reported on the thermal performance of nanofluids in solar collectors. Some of them are related to magnetohydrodynamic transportation and heat transfer properties, while some others are on non-Newtonian fluid flow in porous and asymmetric channels. Cilia-driven flow has not been reported in combination with electromagnetic effects along with heat and mass transfer of SWCNT nanofluids in microchannels for solar thermal applications [9], [10]. Also, the complete study of flow characteristics with the coupled influence of thermal radiation effects on the concentration distribution in electrically conductive SWCNT suspension is not available [11], [12].

A new and complete mathematical model is proposed for the study of peristaltic motion of an electrically conducting SWCNT-based nanofluid in a ciliated microchannel under thermal and electromagnetic fields. The momentum, energy, and concentration equations are solved subject to the assumptions of long wavelength and creeping flow. The present work is original in the sense that it combines cilia-induced pumping with electrically conducting SWCNT nanofluids and coupled electro-thermal phenomena within a single framework that could be of use for systems related to solar thermal energy. Results of such a study could provide a sound theoretical basis for the design and optimization of advanced systems for the conversion of solar energy into other usable forms as well as for thermal management applications.

Transport by cilia in highly constrained spaces is usually treated either by the sublayer model looking at how each cilium actuates in a thin layer or by the envelope model, which examines how closely packed cilia move “synchronously” as a wave. The envelope model works well when the cilia are short compared to the height of the channel and we want a broad sense of the flow generated, or when they are a huge distance away from each other compared to their height.

Figure 1 illustrates the physical configuration and the coordinate system of the problem under consideration. The geometry consists of a two-dimensional asymmetric channel filled with the fluid, where the flow is propelled by the rhythmic beating of cilia on the walls. A Cartesian coordinate system $(\tilde{X}, \tilde{Y})$ is adopted, with the $\tilde{X}$ -axis aligned with the direction of the wave propagation (horizontal) and the $\tilde{Y}$ -axis perpendicular to it (transverse).

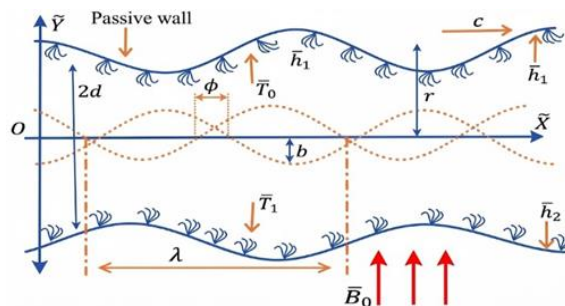


Figure 1. Schematic representation of the geometry of a tapered channel with cilia-driven motion

As explicitly depicted in the figure, the upper and lower walls are situated at $\tilde{Y} = \bar{h}_1$ and $\tilde{Y} = \bar{h}_2$, respectively. The metachronal waves generated by the ciliary motion travel along the channel boundaries with a constant speed c and wavelength λ . To account for the channel's asymmetry, a phase difference ϕ is introduced between the upper and lower waves. Furthermore, the figure demonstrates the applied external

forces and thermal boundary conditions relevant to this study: a uniform transverse magnetic field of strength $\bar{B}_0$ is applied vertically upwards in the $\tilde{Y}$ -direction, and the upper and lower walls are maintained at distinct temperatures, $\bar{T}_0$ and $\bar{T}_1$, respectively. To mathematically characterize such an asymmetric ciliated channel, we adopt the envelope model.

Here we will adopt the envelope model to characterize such an asymmetric ciliated channel. In this viewpoint cilia are depicted by a wave moving on to the walls of the channel and the location of the tips of the cilia effectively sets the edge of the area through which flow is occurring. Thus, the wall geometry can be represented as periodic functions of space and time.

For an asymmetric channel, upper and lower walls are not identical and may be described as [13]:

$$\bar{Y} = \bar{H}_1 = \bar{F}_1(\bar{x}, \bar{t}) = d + \bar{x} \tan \gamma + d \bar{\epsilon} \cos 2 \left[\frac{\phi}{4} + \frac{\pi}{\lambda} (\bar{x} - c\bar{t}) \right], \text{ upper wall,} \quad (1)$$

$$\bar{Y} = \bar{H}_2 = \bar{F}_2(\bar{x}, \bar{t}) = -(d + \bar{x} \tan \gamma) + d \bar{\epsilon} \cos 2 \left[-\frac{\phi}{4} + \frac{\pi}{\lambda} (\bar{x} - c\bar{t}) \right], \text{ lower wall,} \quad (2)$$

where $(\bar{H}_1)$ and $(\bar{H}_2)$ refer to the upper and lower walls, respectively, of the cilia-oriented in a symmetric tapered channel, the parameter γ refers to the angle to which the unperturbed walls of the channel are oriented, encapsulating the tapering geometry of the system. c is the speed at which the metachronal wave travel, and λ refers to its wavelength. The quantity $\bar{\epsilon}$ is a number which shows how long the cilia are in relation to the height of the channel, and then ϕ indicates the lag between waves created on the two channel walls.

In the context of the envelope model, the coordinated beating of the cilia is viewed as a continuous traveling wave along the wall surface. It is assumed the ciliary tips move through an elliptical pathway (representing the horizontal aspect of the power stroke and the vertical aspect of the recovery stroke). Accordingly, the horizontal position of the cilia tips can be mathematically expressed as (3):

$$\bar{x} = \bar{\omega} \bar{\epsilon} d \sin \left[(\bar{x} - c\bar{t}) \frac{2\pi}{\lambda} \right] + \bar{x}_0 = \bar{F}_0(\bar{x}_0, \bar{x}, \bar{t}) \quad (3)$$

The parameters $\bar{\omega}$ and $\bar{x}$ denote the eccentricity parameter of the elliptical trajectory and the reference location of the cilia, respectively. Based on these definitions, the horizontal and vertical components of the ciliary velocity at the position $\bar{x}$ can be expressed as (4) and (5).

$$\bar{U} = \left. \frac{\partial \bar{F}_0}{\partial \bar{t}} \right|_{\bar{x}_0} = \frac{\partial \bar{F}_0}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \bar{t}} + \frac{\partial \bar{F}_0}{\partial \bar{t}} = \frac{\partial \bar{F}_0}{\partial \bar{x}} \bar{U} + \frac{\partial \bar{F}_0}{\partial \bar{t}} \quad (4)$$

$$\left. \begin{aligned} \bar{V} &= \left. \frac{\partial \bar{F}_1}{\partial \bar{t}} \right|_{\bar{x}_0} = \frac{\partial \bar{F}_1}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \bar{t}} + \frac{\partial \bar{F}_1}{\partial \bar{t}} = \frac{\partial \bar{F}_0}{\partial \bar{x}} \bar{U} + \frac{\partial \bar{F}_1}{\partial \bar{t}} \\ \bar{V} &= \left. \frac{\partial \bar{F}_2}{\partial \bar{t}} \right|_{\bar{x}_0} = \frac{\partial \bar{F}_2}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \bar{t}} + \frac{\partial \bar{F}_2}{\partial \bar{t}} = \frac{\partial \bar{F}_0}{\partial \bar{x}} \bar{U} + \frac{\partial \bar{F}_2}{\partial \bar{t}} \end{aligned} \right\} \quad (5)$$

By substituting (3) into (4) and incorporating (1) and (2) into (5), the velocity components can be rewritten in an alternative mathematical form as (6) and (7).

$$\bar{U} = \frac{2ac\bar{\omega}\bar{\epsilon} \pi \cos \left[\frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) \right]}{2ac\bar{\omega}\bar{\epsilon} \pi \cos \left[\frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) \right] - \lambda} \quad (6)$$

$$\bar{V} = \pm \frac{2ac\bar{\omega}\bar{\epsilon} \pi \left(\sin \left[\frac{\phi}{4} + \frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) \right] + \bar{\omega} \cos \left[\frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) \right] \tan \gamma \right)}{2ac\bar{\omega}\bar{\epsilon} \pi \cos \left[\frac{2\pi}{\lambda} (\bar{x} - c\bar{t}) \right] - \lambda} \quad (7)$$

2. RESEARCH METHOD

2.1. Basic equation

The governing equations describing the flow under the specified conditions are presented below, starting with the continuity equation:

$$\frac{\partial \bar{U}}{\partial \bar{x}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \quad (8)$$

Motion equations: the $(\bar{x})$ component of moment equation:

$$\rho_{nf} \left(\frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{Y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{X}} + \mu_{nf} \left(\frac{\partial^2 \bar{U}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{U}}{\partial \bar{Y}^2} \right) - \sigma B_0^2 \bar{U} + \bar{\rho} e E \bar{X} + (\rho \gamma)_{nf} g(T - T_0) + g p B_c (C - C_0) \quad (9)$$

The $\bar{y}$ -component of moment equation:

$$(\rho c_p)_{nf} \left(\frac{\partial \bar{V}}{\partial t} + \bar{U} \frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{V}}{\partial \bar{Y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{Y}} + \mu_{nf} \left(\frac{\partial^2 \bar{V}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{V}}{\partial \bar{Y}^2} \right) + \bar{\rho} e E \bar{Y} \quad (10)$$

The energy equation:

$$(\rho c_p)_f \left(\frac{\partial \bar{T}}{\partial t} + \bar{U} \frac{\partial \bar{T}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{T}}{\partial \bar{Y}} \right) = k \left(\frac{\partial^2 \bar{T}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{Y}^2} \right) + (\rho c_p)_p \frac{D_T}{T_m} \left[\left(\frac{\partial \bar{T}}{\partial \bar{X}} \right)^2 + \left(\frac{\partial \bar{T}}{\partial \bar{Y}} \right)^2 \right] \quad (11)$$

The concentration equation:

$$\left(\frac{\partial \bar{C}}{\partial t} + \bar{U} \frac{\partial \bar{C}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{C}}{\partial \bar{Y}} \right) = D_m \left(\frac{\partial^2 \bar{C}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{Y}^2} \right) + \bar{k} r (C - C_0) + \frac{D_m k_T}{T_m} \left(\frac{\partial^2 \bar{T}}{\partial \bar{X}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{Y}^2} \right) \quad (12)$$

In the above expressions, $\bar{p}$ represents the pressure, while $(\rho \gamma)_{nf}$ denotes the effective thermal expansion coefficient of the nanofluid. The parameter ρ_{nf} refers to the effective density of the nanofluid, and $\bar{\rho} e$ represents the charge number density. The quantities $E \bar{X}$ and $E \bar{Y}$ indicate the electrokinetic body forces acting in the $\bar{x}$ and $\bar{y}$ directions, respectively. In addition, μ_{nf} denotes the effective thermal diffusivity of the nanofluid, whereas $\bar{T}$ represents the temperature of the fluid.

The thermal conductivity of the carbon nanotube suspension is described using the Xue model. This model is commonly employed to evaluate the heat transfer characteristics of nanofluids containing carbon nanotubes. It highlights the improved thermophysical properties of SWCNT water nanofluids, as discussed in reference [14].

$$\left. \begin{aligned} \rho_{nf} &= (1 - \phi) \rho_b + \phi \rho_{SWCNT}, \mu_{nf} = \frac{\mu_b}{(1 - \phi)^{2.5}} \\ (\rho c_p)_{nf} &= (1 - \phi) (\rho c_p)_b + \phi (\rho c_p)_{SWCNT}, \alpha_{nf} = \frac{K_{nf}}{(\rho c_p)_{nf}}, \\ K_{nf} &= \frac{(1 - \phi) + 2\phi \frac{K_{SWCNT}}{K_b} \ln \left(\frac{K_{SWCNT} + K_b}{2K_b} \right)}{(1 - \phi) + 2\phi \frac{K_b}{K_{SWCNT} - K_b} \ln \left(\frac{K_{SWCNT} + K_b}{2K_b} \right)} \end{aligned} \right\} \quad (13)$$

where ρ_b is the density of the base fluid (water), ρ_{SWCNT} is the density of SWCNTs, K_b is the thermal conductivity of water, K_{SWCNT} is the thermal conductivity of SWCNTs, and ϕ represented the value of nanoparticle volume fraction, which expresses the amount of SWCNT dispersed in the base fluid.

Electrohydrodynamics or electrokinetics is the study of behavior of electrically charged fluids and fluid motion due to ions immersed in electric fields. In systems with electrolytes having charged surfaces, an electrical double layer (EDL) forms, where positive and negative ions separate near the solids-liquid boundary. The distribution of the electrical potential structure within this region gives rise to an electrostatic body force acting on the fluid.

To describe the spatial variation of the electric potential $\bar{\Phi}$ across the EDL, the Poisson equation is:

$$\nabla^2 \bar{\Phi} = -\frac{\bar{\rho} e}{\epsilon_0} \quad (14)$$

Where $\bar{\rho} e$ is the net charge density of the fluid and ϵ is the dielectric permittivity of the medium. One of the important relationships that arises from this is Gauss' law [15].

Here, $\bar{\rho} e$ denotes the electric charge number density, which is expressed as (15):

$$\bar{\rho} e = ez (\bar{n}^+ - \bar{n}^-) \quad (15)$$

where e denotes the elementary electric charge and z represents the valence (charge number) of the ionic species. The quantities $\bar{n}^+$ and $\bar{n}^-$ correspond to the number densities of positive cations and negative anions, respectively. The distribution of electric potential within the system is therefore governed by the local charge number density. Accordingly, the spatial distribution of ions in the fluid can be described using the Nernst-Planck equation [16], which is expressed as (16):

$$\left(\frac{\partial \bar{n}^{\pm}}{\partial \bar{t}} + \bar{U} \cdot \frac{\partial \bar{n}^{\pm}}{\partial \bar{x}} + \bar{V} \cdot \frac{\partial \bar{n}^{\pm}}{\partial \bar{y}} \right) = D_m \left(\frac{\partial^2 \bar{n}^{\pm}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{n}^{\pm}}{\partial \bar{y}^2} \right) \pm \frac{D_m e z}{T_e K_B} \left[\frac{\partial}{\partial \bar{x}} \left(\bar{n}^{\pm} \frac{\partial \bar{\kappa}}{\partial \bar{x}} \right) + \frac{\partial}{\partial \bar{y}} \left(\bar{n}^{\pm} \frac{\partial \bar{\kappa}}{\partial \bar{y}} \right) \right] \quad (16)$$

where K_B denotes the Boltzmann constant, $\bar{T}_e$ represents the mean temperature of the electrolyte solution, and D_m corresponds to the mass diffusion coefficient.

In the fixed laboratory coordinate system $(\bar{x}, \bar{y})$, the flow exhibits an unsteady behavior because of the propagation of the wave. However, when the analysis is performed in a reference frame moving with the wave at a constant velocity c , the flow appears steady in that moving coordinate system.

$$\left. \begin{aligned} \bar{X} &= \bar{x} - c\bar{t}, \bar{Y} = \bar{y}, \bar{U}(\bar{X}, \bar{Y}, \bar{t}) = \bar{u}(\bar{x}, \bar{y}) - c, \\ \bar{V}(\bar{X}, \bar{Y}, \bar{t}) &= \bar{v}(\bar{x}, \bar{y}), \bar{P}(\bar{X}, \bar{Y}, \bar{t}) = \bar{p}(\bar{x}, \bar{y}), \\ \bar{T}(\bar{X}, \bar{Y}, \bar{t}) &= \bar{T}(\bar{x}, \bar{y}), \bar{C}(\bar{X}, \bar{Y}, \bar{t}) = \bar{C}(\bar{x}, \bar{y}) \end{aligned} \right\} \quad (17)$$

where $\bar{u}$, $\bar{T}$, $\bar{C}$, and $\bar{p}$ are the velocity components, the temperature, the concentration, and the pressure in the wave frame, respectively. Thereafter, transform (1), (2), and (8) up to (12) into the wave frame using (14) as specified in the prior expressions.

$$\bar{y} = \bar{H}_1 = d + \bar{x} \tan \gamma + d \bar{\epsilon} \cos\left(\frac{2\pi}{\lambda} \bar{x}\right) \quad (18)$$

upper wall,

$$\bar{y} = \bar{H}_2 = -(d + \bar{x} \tan \gamma) + d \bar{\epsilon} \cos\left(\frac{2\pi}{\lambda} \bar{x}\right) \quad (19)$$

lower wall.

$$\frac{\partial(\bar{u}-c)}{\partial(\bar{x}-c\bar{t})} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (20)$$

$$\rho_{nf} \left((\bar{u}-c) \frac{\partial(\bar{u}-c)}{\partial(\bar{x}-c\bar{t})} + \bar{v} \frac{\partial(\bar{u}-c)}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial(\bar{x}-c\bar{t})} + \mu_{nf} \left(\frac{\partial^2(\bar{u}-c)}{\partial(\bar{x}-c\bar{t})^2} + \frac{\partial^2(\bar{u}-c)}{\partial \bar{y}^2} \right) - \sigma B_0^2 (\bar{u}-c) + \left[-\epsilon_0 \left(\frac{\partial^2 \bar{\kappa}}{\partial(\bar{x}-c\bar{t})^2} + \frac{\partial^2 \bar{\kappa}}{\partial \bar{y}^2} \right) \right] E \bar{x} + (\rho \gamma)_{nf} g(T - T_0) + g p B_c (C - C_0), \quad (21)$$

$$(\rho c)_{nf} \left((\bar{u}-c) \frac{\partial \bar{v}}{\partial(\bar{x}-c\bar{t})} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu_{nf} \left(\frac{\partial^2 \bar{v}}{\partial(\bar{x}-c\bar{t})^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right) \left[-\epsilon_0 \left(\frac{\partial^2 \bar{\kappa}}{\partial(\bar{x}-c\bar{t})^2} + \frac{\partial^2 \bar{\kappa}}{\partial \bar{y}^2} \right) \right] E \bar{y}, \quad (22)$$

$$(\rho c)_f \left((\bar{u}-c) \frac{\partial \bar{T}}{\partial(\bar{x}-c\bar{t})} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = k \left(\frac{\partial^2 \bar{T}}{\partial(\bar{x}-c\bar{t})^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right) + (\rho c_p)_b \frac{D_T}{T_m} \left[\left(\frac{\partial \bar{T}}{\partial(\bar{x}-c\bar{t})} \right)^2 + \left(\frac{\partial \bar{T}}{\partial \bar{y}} \right)^2 \right], \quad (23)$$

$$\left((\bar{u}-c) \frac{\partial \bar{C}}{\partial(\bar{x}-c\bar{t})} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} \right) = D_m \left(\frac{\partial^2 \bar{C}}{\partial(\bar{x}-c\bar{t})^2} + \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \right) + \bar{\kappa} (C - C_0) + \frac{D_m k_T}{T_m} \left(\frac{\partial^2 \bar{T}}{\partial(\bar{x}-c\bar{t})^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right), \quad (24)$$

2.2. Dimensionless quantities

The subsequent non-dimensional quantities were established to conduct the non-dimensional analysis:

$$\left. \begin{aligned} x &= \frac{1}{\lambda} \bar{x}, y = \frac{1}{d} \bar{y}, u = \frac{1}{c} \bar{u}, v = \frac{1}{\delta c} \bar{v}, p = \frac{d^2}{\lambda \mu_b c} \bar{p}, t = \frac{c}{\lambda} \bar{t}, h_1 = \frac{1}{d} \bar{H}_1, h_2 = \frac{1}{d} \bar{H}_2 \\ \delta &= \frac{d}{\lambda}, \emptyset = \frac{C-C_0}{C_1-C_0}, M^2 = \frac{\sigma d^2 \beta_0^2}{\mu_b}, Uhs = \frac{\epsilon K_b \hat{T} Ex}{ez \mu_b c}, \bar{T} = T - T_0, \theta = \frac{T-T_0}{T_1-T_0}, \\ Gc &= \frac{P_g B_c d^2 (C_1 - C_0)}{\mu_c}, Ec = \frac{c^2}{c_p (T_1 - T_0)}, Sc = \frac{\mu}{\rho D_m}, kr = \frac{d}{c} \bar{\kappa}, Pr = \frac{\mu c_p}{k}, \\ Gr &= \frac{P_b g \gamma_b d^2 (T_1 - T_0)}{\mu_b c}, Sr = \frac{\rho D_m (T_1 - T_0) K_T}{\mu T_m (C_1 - C_0)}, \mathfrak{K} = \frac{ez}{K_b \bar{T}} \bar{\kappa}, L = \frac{(\rho \gamma)_{nf}}{(\rho \gamma)_f}, \\ (\rho c_p)_f &= \frac{\mu_{nf}}{v}, \hat{\tau} = \frac{(\rho c_p)_b}{(\rho c_p)_f}, Nt = \frac{\hat{\tau} D_T (T_1 - T_0)}{T_m v}, Re = \frac{(\rho c)_f c d}{\mu_{nf}}, n = \frac{\bar{n}}{n_0} \end{aligned} \right\} \quad (25)$$

In the above formulation, T_1 and T_0 represent the dimensional temperatures of the fluid and the reference state, respectively, while C_1 and C_0 denote the corresponding dimensional concentrations.

F is the instantaneous volumetric flow rate (dimensionless), Q is the mean volumetric flow rate over one wavelength (dimensionless). The parameter δ is the wave number (dimensionless) (generally taken as the

ratio of the height of the channel to the wavelength. If it is small, the approximation of long wavelengths is used; Uhs is the non-dimensional channel wall transverse displacement [16]–[18].

The significance of the non-dimensional parameters appearing in the model are as follows: Nt is known as the Thermophoresis number which characterizes the migration of the nanoparticles as a result of temperature gradients, kr is the chemical reaction parameter representative of the strength of the chemical reaction taking place in the flow, Pr is the Prandtl number, referred to as the ratio of the momentum diffusivity to the thermal diffusivity, Sr is the Soret number which indicates the effect of temperature differences across the liquid mass causing mass to flow along with it i.e., the thermal diffusion effect [19]–[21].

Gr is the Grashof number, representing the relative influence of the buoyancy forces to the viscous forces. Ec is the Eckert number, relating viscous dissipation to the imposed temperature difference [22]–[24]. Sc is the Schmidt number (momentum diffusivity/mass diffusivity). Gc is the mass flux number (buoyancy altered by concentrations being different). M is the Hartmann number (viscous/magnetic). Here, ϕ is the non-dimensional nanoparticle concentration and θ is the non-dimensional temperature. These dimensionless numbers govern the coupled thermal, concentration, magnetic, and reactive transport of a nanofluid in the system of two asymmetric ciliated channels. Then, in view of (25), (18) to (24) take the form:

$$h_1 = 1 + \frac{1}{\delta} x \tan \gamma + \epsilon \cos [2x]; \quad (26)$$

$$h_2 = -1 - \frac{1}{\delta} x \tan \gamma + \epsilon \cos [2x]; \quad (27)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (28)$$

$$\left((1 - \phi) \rho_b \frac{c d}{\mu_b} + \phi \rho_{\text{SWCNT}} \frac{c d}{\mu_b} \right) \left((u - 1) \delta \frac{\partial u}{\partial x} + \delta v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \frac{1}{(1 - \phi)^{2.5}} \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - M^2 (u - 1) - Uhs \left(\delta^2 \frac{\partial^2 \kappa}{\partial x^2} + \frac{\partial^2 \kappa}{\partial y^2} \right) + \frac{(PY)_{nf}}{(PY)_f} \frac{P_b Y_b g d^2 (T_1 - T_0)}{\mu_b c} + Gc \phi \quad (29)$$

$$(\rho_c)_{nf} \frac{d^2 c}{\mu_b} \delta^2 \left((u - 1) \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\mu_b} \left(\delta^4 \frac{\partial^2 v}{\partial x^2} + \delta^2 \frac{\partial^2 v}{\partial y^2} \right) + Uhs \delta \left(\delta^2 \frac{\partial^2 \kappa}{\partial x^2} + \frac{\partial^2 \kappa}{\partial y^2} \right), \quad (30)$$

$$\text{Re} \delta \left((u - 1) \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \frac{1}{\text{Pr}} \left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \text{Nt} \left[\delta^2 \left(\frac{\partial \theta}{\partial x} \right)^2 + \left(\frac{\partial \theta}{\partial y} \right)^2 \right], \quad (31)$$

$$\delta \left((u - 1) \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} \right) = \frac{1}{Sc} \left(\delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + kr \phi + Sr \left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right), \quad (32)$$

2.3. The problem's solution

The incorporation of the dimensionless stream function (ψ) via the relations $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ in (26)–(32) illustrates that the continuity equation is satisfied identically, whereas the remaining equations, under the long wavelength approximation (i.e., disregarding wave number ($\delta \ll 1$) and Reynolds number ($\text{Re} \ll 1$)),

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0, \quad (33)$$

$$\frac{\partial p}{\partial x} = \frac{1}{(1 - \phi)^{2.5}} \left(\frac{\partial^3 \psi}{\partial y^3} \right) - M^2 \left(\frac{\partial \psi}{\partial y} - 1 \right) - Uhs \left(\frac{\partial^2 \kappa}{\partial y^2} \right) + L Gr \theta + Gc \phi \quad (34)$$

By differentiating both sides of (34) with respect to y,

$$\frac{1}{(1 - \phi)^{2.5}} \left(\frac{\partial^4 \psi}{\partial y^4} \right) - M^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right) - Uhs \left(\frac{\partial^3 \kappa}{\partial y^3} \right) + L Gr \left(\frac{\partial \theta}{\partial y} \right) + Gc \left(\frac{\partial \phi}{\partial y} \right) = 0, \quad (35)$$

$$\frac{\partial p}{\partial y} = 0 \quad (36)$$

$$\left(\frac{\partial^2 \theta}{\partial y^2} \right) + \text{Nt Pr} \left(\frac{\partial \theta}{\partial y} \right)^2 = 0, \quad (37)$$

$$\frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + kr \phi + Sr \frac{\partial^2 \theta}{\partial y^2} = 0, \quad (38)$$

$$\frac{\partial^2 \kappa}{\partial y^2} = k^2 \left(\frac{\bar{n}^- - \bar{n}^+}{2} \right), \quad (39)$$

$$\frac{\partial^2 \bar{n}^\pm}{\partial y^2} = \pm \frac{\partial}{\partial y} \left(\bar{n}^\pm \frac{\partial \kappa}{\partial y} \right), \quad (40)$$

By applying the boundary condition to (40), it results in:

$$\bar{n}^\pm = e^{\pm \kappa} \quad (41)$$

The Poisson-Boltzmann paradigm is derived by combining (39) and (41):

$$\frac{\partial^2 \kappa}{\partial y^2} = k^2 \sinh(\kappa) \quad (42)$$

The Debye-Hückel approximation method, which presumes a diminished zeta potential within the EDL, can be utilized to reduce the specified equation as (43):

$$\frac{\partial^2 \kappa}{\partial y^2} = k^2 \kappa \quad (43)$$

From (43) is integrated directly while applying the following boundary conditions:

$$\kappa|_{y=h} = 1, \quad \left. \frac{\partial \kappa}{\partial y} \right|_{y=0} = 0 \quad (44)$$

and the resultant electric potential function is expressed as (45).

$$\kappa = \frac{\cosh(ky)}{\cosh(kh)}, \quad (45)$$

2.4. Stream function solution

By incorporating (45) into (34) and (35) we obtain:

$$\frac{\partial p}{\partial x} = \frac{1}{(1-\phi)^{2.5}} \left(\frac{\partial^3 \psi}{\partial y^3} \right) - M^2 \left(\frac{\partial \psi}{\partial y} - 1 \right) - Uhs k^2 \frac{\cosh(ky)}{\cosh(kh)} + L Gr \theta + Gc \phi \quad (46)$$

$$\frac{1}{(1-\phi)^{2.5}} \left(\frac{\partial^4 \psi}{\partial y^4} \right) - M^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right) - Uhs k^3 \frac{\sinh(ky)}{\cosh(kh)} + L Gr \left(\frac{\partial \theta}{\partial y} \right) + Gc \left(\frac{\partial \phi}{\partial y} \right) = 0 \quad (47)$$

In the wave framework, the dimensionless boundary conditions are [25]:

$$\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1 \text{ at } y = h_1, \quad \psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1 \text{ at } y = h_2 \quad (48)$$

$$\psi = \frac{\frac{g_1 y^2}{2} + \frac{1}{2} g_3 Gc y^2 + \frac{1}{2} g_2 Gr Ly^2 + e^{\frac{My}{\sqrt{w}}} w A1 + e^{-\frac{My}{\sqrt{w}}} w A2}{M^2} + A3 + y A4; \quad (49)$$

where $w = \frac{1}{(1-\phi)^{2.5}}$, $g_1 = Uhs k^3 \frac{\sinh(ky)}{\cosh(kh)}$, $g_2 = \frac{\partial \theta}{\partial y}$, $g_3 = \frac{\partial \phi}{\partial y}$, and the constants are A1, A2, A3, and A4.

2.5. Solution to the energy equation

Due to the nonlinear characteristics of (37), deriving a closed-form solution for all arbitrary values is impractical. Utilized the perturbation method to determine the solution, elaborate the perturbation solution, and analyze the perturbations:

$$\theta = \theta_0 + Pr \theta_1 + O((Pr)^2), \quad (50)$$

2.5.1. Zeroth order

When the order terms (Pr) are insignificant in the zeroth order system, we obtain:

$$\frac{\partial^2 \theta_0}{\partial y^2} = 0 \quad (51)$$

2.5.2. First-order solution

$$\left(\frac{\partial^2 \theta_1}{\partial y^2}\right) + Nt \left(\frac{\partial \theta_0}{\partial y}\right)^2 = 0 \quad (52)$$

The boundary conditions are given in [10]:

$$\theta_0 = 0 \text{ at } y = h_1, \theta_1 = 0 \text{ at } y = h_1, \theta_0 = 1 \text{ at } y = h_2, \theta_1 = 0 \text{ at } y = h_2 \quad (53)$$

$$\left. \begin{aligned} \theta_0 &= \frac{h_1}{h_1-h_2} - \frac{y}{h_1-h_2} \\ \theta_1 &= \left(-\frac{-2h_1^2+2h_1h_2+h_1h_2Nt}{2(h_1-h_2)^2} - \frac{(2h_1-2h_2-h_1Nt-h_2Nt)y}{2(h_1-h_2)^2} - \frac{Nty^2}{2(h_1-h_2)^2} \right) \end{aligned} \right\} \quad (54)$$

$$\theta = \frac{h_1}{h_1-h_2} - \frac{y}{h_1-h_2} + pr \left(-\frac{-2h_1^2+2h_1h_2+h_1h_2Nt}{2(h_1-h_2)^2} - \frac{(2h_1-2h_2-h_1Nt-h_2Nt)y}{2(h_1-h_2)^2} - \frac{Nty^2}{2(h_1-h_2)^2} \right) \quad (55)$$

2.6. Solution to the concentration equation

Solve the concentration equation from (38) by applying the following boundary conditions:

$$\phi = 0 \text{ at } y = h_1, \phi = 1 \text{ at } y = h_2 \quad (56)$$

$$\phi = \frac{NtprSr}{(h_1-h_2)^2kr} + b_1\cos[\sqrt{kr}\sqrt{Sc}y] + b_2\sin[\sqrt{kr}\sqrt{Sc}y] \quad (57)$$

$$b_1 = -\frac{\left(-1+\frac{NtprSr}{(h_1-h_2)^2kr}\right)\sin[h_1\sqrt{kr}\sqrt{Sc}]-\frac{NtprSr\sin[h_2\sqrt{kr}\sqrt{Sc}]}{(h_1-h_2)^2kr}}{\cos[h_2\sqrt{kr}\sqrt{Sc}]\sin[h_1\sqrt{kr}\sqrt{Sc}]-\cos[h_1\sqrt{kr}\sqrt{Sc}]\sin[h_2\sqrt{kr}\sqrt{Sc}]} \quad (58)$$

$$\begin{aligned} b_2 &= (-h_1^2kr\cot[h_1\sqrt{kr}\sqrt{Sc}] + 2h_1h_2kr\cot[h_1\sqrt{kr}\sqrt{Sc}] - h_2^2kr\cot[h_1\sqrt{kr}\sqrt{Sc}] + \\ &NtprSr\cot[h_1\sqrt{kr}\sqrt{Sc}] - NtprSr\cos[h_2\sqrt{kr}\sqrt{Sc}]\csc[h_1\sqrt{kr}\sqrt{Sc}]) / ((h_1 - \\ &h_2)^2kr(\cos[h_2\sqrt{kr}\sqrt{Sc}] - \cot[h_1\sqrt{kr}\sqrt{Sc}]\sin[h_2\sqrt{kr}\sqrt{Sc}])) \end{aligned} \quad (59)$$

3. RESULTS AND DISCUSSION

To study the effects of governing physical parameters collar (Uhs), M, Nt, γ of the unperturbed walls, Pr, Gr, kr, Sr, Ec, Gc, and Sc, an in-depth parametric study was performed. The effects of these parameters on the pressure gradient (dp/dx), axial velocity (u), and temperature (θ) are examined below. All calculations and plots were generated using Mathematica.

3.1. Pressure gradient dp/dx

Figures 2–7 show how the main governing parameters affect dp/dx. One sees from the graphical results how the various dimensionless numbers modify the axial pressure distribution over the width of the channel.

Figures 2, 5, and 6 show that as Gr, kr, and Sc increase the pressure gradient at first, especially towards the left side of moving toward the channel center, the tendency is for its magnitude to decay continuously, before becoming nearly negligible at the right wall. This signifies now that buoyancy forces, heat spreading, chemical reactions, and how easily substances mix all contribute to the resistance to flow at the beginning of the channel, but effectively weaker as you move further.

Figure 3 shows M effect on the pressure gradient. The higher M, the higher the pressure gradient, especially towards the middle of the channel. However, it is clear that the influence of the magnetic field starts to lessen as one approaches the right wall, tending to an almost neutrally buoyant state pressure gradient towards that wall. There exists an inflection point between the left wall and the center of the channel, indicating that at that region of the channel, one has a lower pressure gradient, before it starts to increase again as one approaches the middle of the channel.

Figure 4 displays the effect of pressure gradient on transverse wall displacement parameter Uhs. As Uhs increases, the pressure gradient decreases markedly between the left wall and the channel center, but beyond the mid-channel it tends to diminish and become insignificant towards the right wall. This indicates that the wall deformation affects the local flow resistance predominantly over the upstream half of the channel.

Finally, the trend with increasing Sr is shown in Figure 7. An increase of Sr corresponds to a decrease of the pressure difference from the left boundary to the center of the channel, with no apparent

change on the right side. This shows that thermal diffusion influences only how the buoyancy from concentration affects what happens in the upstream region, with little effect on the pressure from the downstream side. The graphical analysis shows that all dimensionless parameters vary the pressure gradient with different non-uniformities through space. The highest sensitivity is often found near the left boundary and the mid-point of the asymmetric channel.

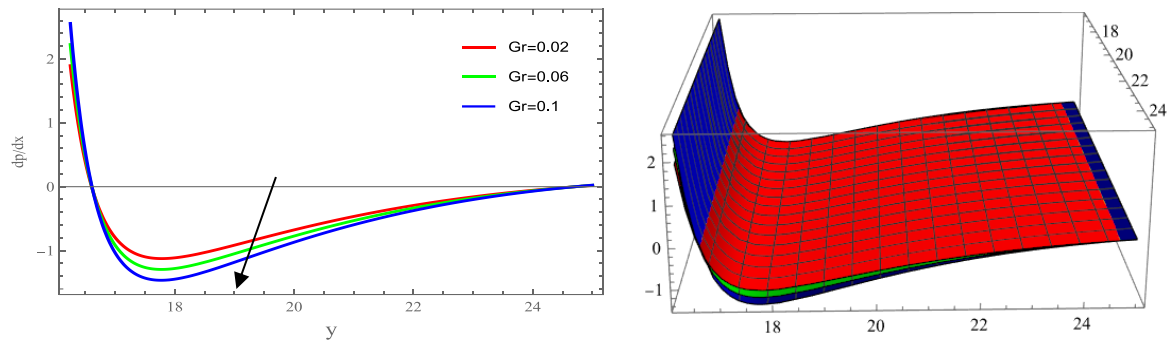


Figure 2. Variation of pressure gradient dp/dx for different values of Gr when $L=0.2$, $M=0.08$, $Nt=0.5$, $Pr=0.4$, $\epsilon=0.02$, $K=0.08$, $U_{hs}=0.02$, $Kr=0.08$, $Sc=0.02$, $Q=2$, $Gc=0.02$, $h=2$, $Sr=0.02$, $\delta=0.001$, $\gamma=0.03$, $d=1$, and $x=1$

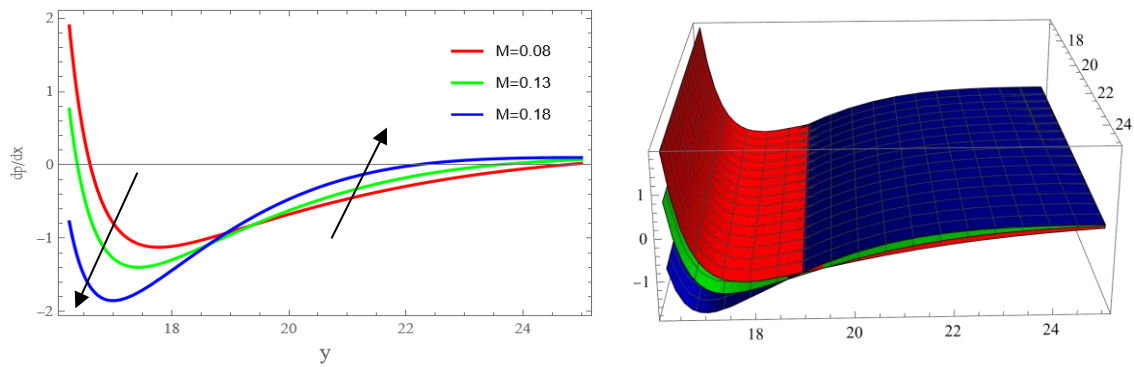


Figure 3. Variation of pressure gradient dp/dx for different values of M when $L=0.2$, $Gr=0.02$, $Nt=0.5$, $Pr=0.4$, $\epsilon=0.02$, $K=0.08$, $U_{hs}=0.02$, $Kr=0.08$, $Sc=0.02$, $Q=2$, $Gc=0.02$, $h=2$, $Sr=0.02$, $\delta=0.001$, $\gamma=0.03$, $d=1$, and $x=1$

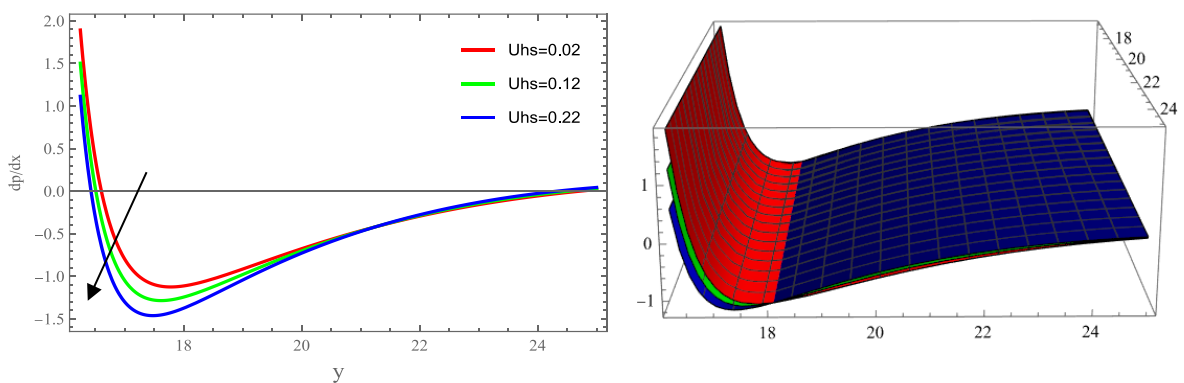


Figure 4. Variation of pressure gradient dp/dx for different values of U_{hs} when $L=0.2$, $Gr=0.02$, $M=0.08$, $Nt=0.5$, $\epsilon=0.02$, $K=0.08$, $Pr=0.4$, $Kr=0.08$, $Sc=0.02$, $Q=2$, $Gc=0.02$, $h=2$, $Sr=0.02$, $\delta=0.001$, $\gamma=0.03$, $d=1$, and $x=1$

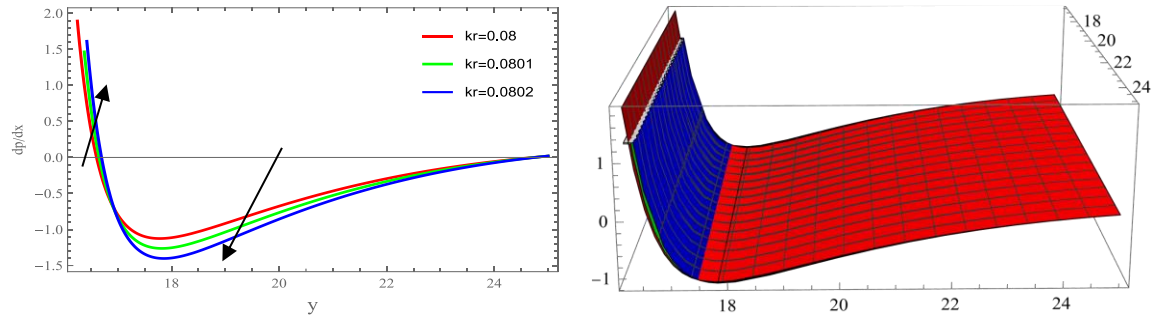


Figure 5. Variation of pressure gradient dp/dx for different values of Kr when $L=0.2$, $Gr=0.02$, $M=0.08$, $Nt=0.5$, $\epsilon=0.02$, $K=0.08$, $Pr=0.4$, $Uhs=0.02$, $Sc=0.02$, $Q=2$, $Gc=0.02$, $h=2$, $Sr=0.02$, $\delta=0.001$, $\gamma=0.03$, $d=1$, and $x=1$

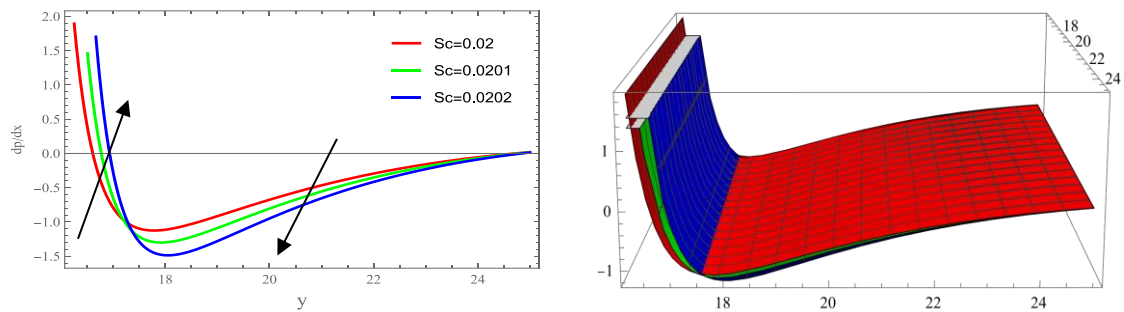


Figure 6. Variation of pressure gradient dp/dx for different values of Sc when $L=0.2$, $Gr=0.02$, $M=0.08$, $Nt=0.5$, $\epsilon=0.02$, $K=0.08$, $Pr=0.4$, $Uhs=0.02$, $Kr=0.08$, $Q=2$, $Gc=0.02$, $h=2$, $Sr=0.02$, $\delta=0.001$, $\gamma=0.03$, $d=1$, and $x=1$

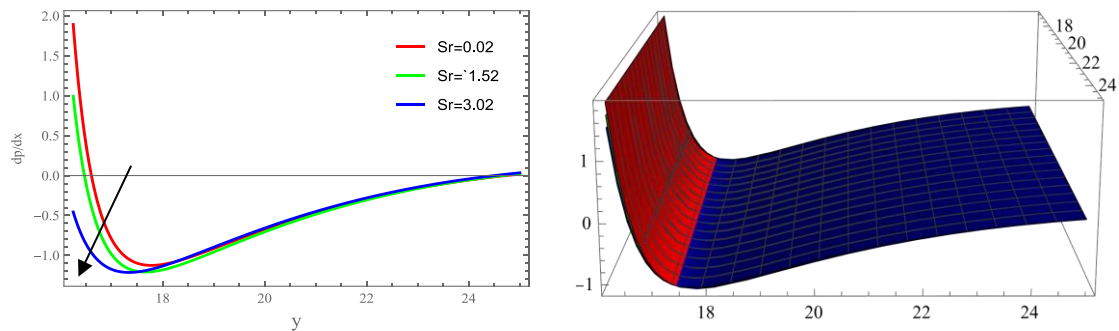


Figure 7. Variation of pressure gradient dp/dx for different values of Sr when $L=0.2$, $Gr=0.02$, $M=0.08$, $Nt=0.5$, $\epsilon=0.02$, $K=0.08$, $Pr=0.4$, $Uhs=0.02$, $Kr=0.08$, $Q=2$, $Gc=0.02$, $h=2$, $Sc=0.02$, $\delta=0.001$, $\gamma=0.03$, $d=1$, and $x=1$

3.2. Velocity u

Figures 8–14 indicate the effects of the governing dimensionless parameters on the axial velocity u . They depict the resulting changes on the flow structure in the asymmetric channel due to varying the thermal, magnetic, concentration, and geometrical effects. Figures 8 and 10–14 show that as the size of axial velocity ($|u|$) increase, so does value of Gr , Uhs , kr , Sc , Gc , and Sr . This increase signifies that the factors of buoyancy, heat spreading, concentration differences and wall moving means are working together to make the fluid move faster. Higher Gr and Gc make the buoyancy-driven flow stronger and Uhs make the shape of the channel in a way that suits the fluid.

As shown in Figure 9, the absolute velocity decreases with increasing M as a stronger magnetic field increases the Lorentz force resisting the motion of the fluid. In general, these considerations of velocity indicate that the effects of buoyancy and wall deformation tend to produce acceleration of the fluid, while those of magnetism and thermophoresis tend to produce a reduction in the velocity through the channel.

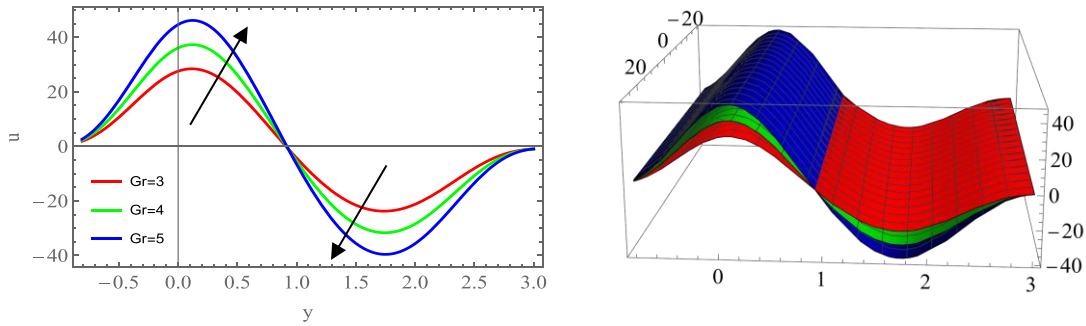


Figure 8. Variation of velocity u for different values of Gr when $M=0.2$, $Nt=0.05$, $Pr=0.2$, $\epsilon=0.02$, $\delta=0.001$, $k=0.02$, $U_{hs}=2$, $Kr=0.3$, $Sc=3$, $Q=2$, $Gc=0.04$, $h=1.2$, $Sr=0.3$, $\gamma=0.002$, $d=2$, and $x=1$

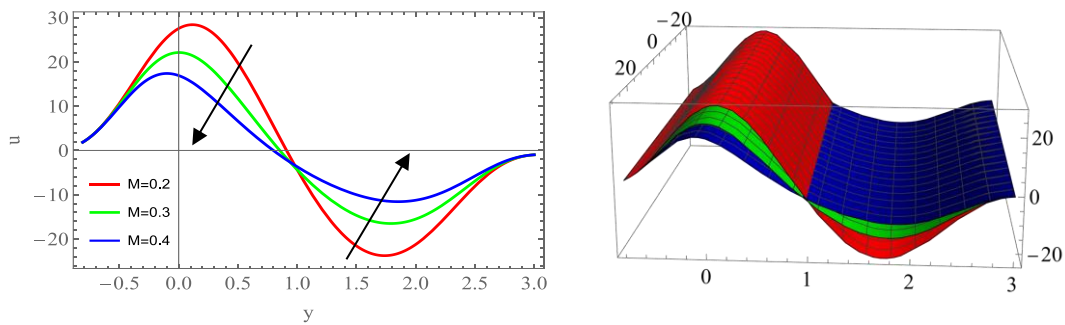


Figure 9. Variation of velocity u for different values of M when $Gr=3$, $Nt=0.05$, $Pr=0.2$, $\epsilon=0.02$, $\delta=0.001$, $k=0.02$, $U_{hs}=2$, $Kr=0.3$, $Sc=3$, $Q=2$, $Gc=0.04$, $h=1.2$, $Sr=0.3$, $\gamma=0.002$, $d=2$, and $x=1$

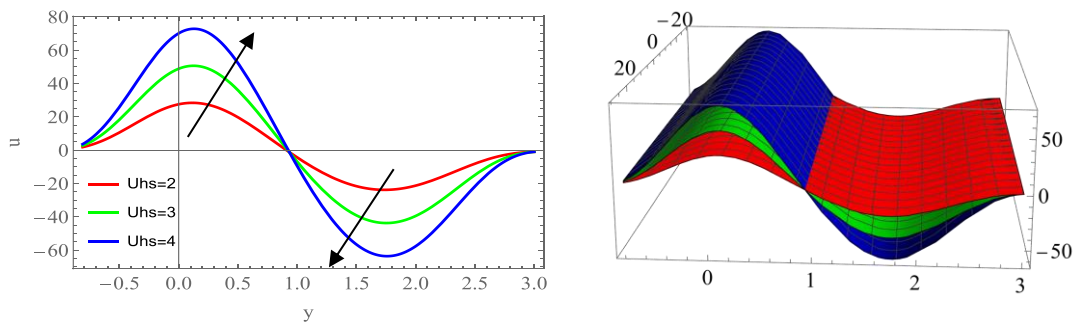


Figure 10. Variation of velocity u for different values of U_{hs} when $Gr=3$, $Pr=0.02$, $M=0.2$, $\epsilon=0.02$, $\delta=0.001$, $k=0.02$, $Nt=0.05$, $Kr=0.3$, $Sc=3$, $Q=2$, $Gc=0.04$, $h=1.2$, $Sr=0.3$, $\gamma=0.002$, $d=2$, and $x=1$

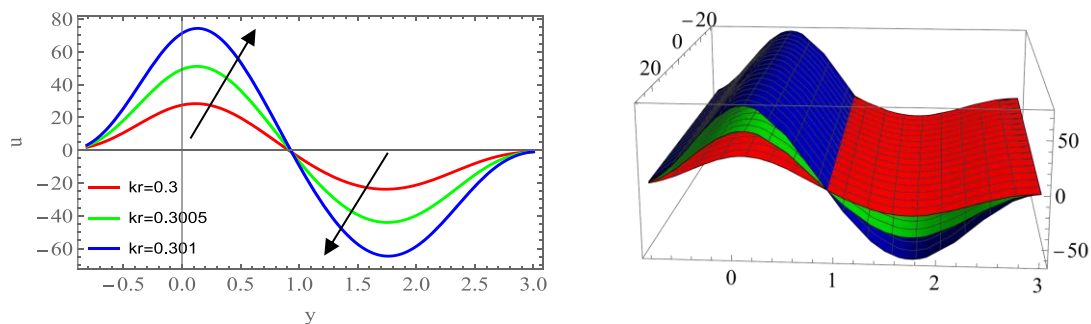


Figure 11. Variation of velocity u for different values of Kr when $Gr=3$, $Pr=0.02$, $M=0.2$, $\epsilon=0.02$, $\delta=0.001$, $k=0.02$, $Nt=0.05$, $U_{hs}=2$, $Sc=3$, $Q=2$, $Gc=0.04$, $h=1.2$, $Sr=0.3$, $\gamma=0.002$, $d=2$, and $x=1$

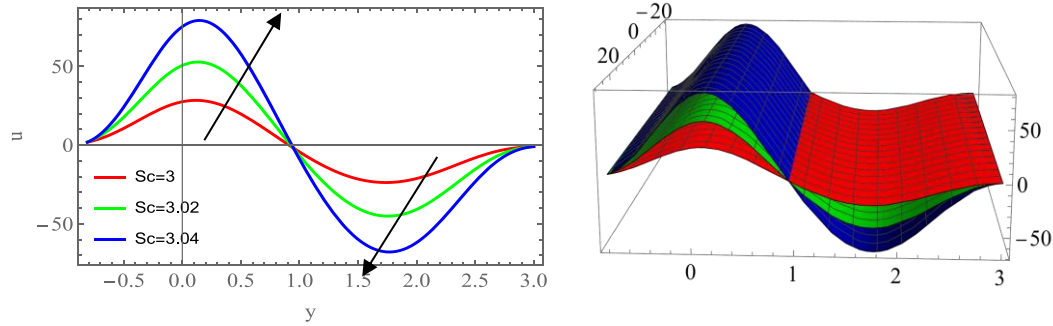


Figure 12. Variation of velocity u for different values of Sc when $Gr=3$, $Pr=0.02$, $M=0.2$, $\epsilon=0.02$, $\delta=0.001$, $k=0.02$, $Nt=0.05$, $U_{hs}=2$, $Kr=0.3$, $Q=2$, $Gc=0.04$, $h=1.2$, $Sr=0.3$, $\gamma=0.002$, $d=2$, and $x=1$

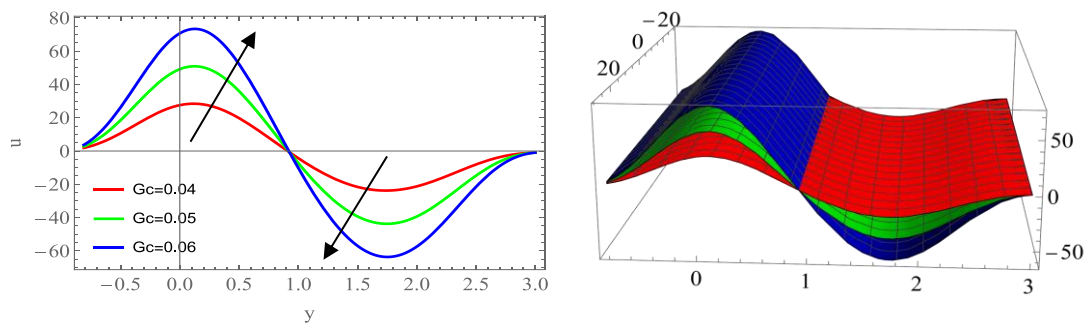


Figure 13. Variation of velocity u for different values of Gc when $Gr=3$, $Pr=0.02$, $M=0.2$, $\epsilon=0.02$, $\delta=0.001$, $k=0.02$, $Nt=0.05$, $U_{hs}=2$, $Kr=0.3$, $Q=2$, $Sc=3$, $h=1.2$, $Sr=0.3$, $\gamma=0.002$, $d=2$, and $x=1$

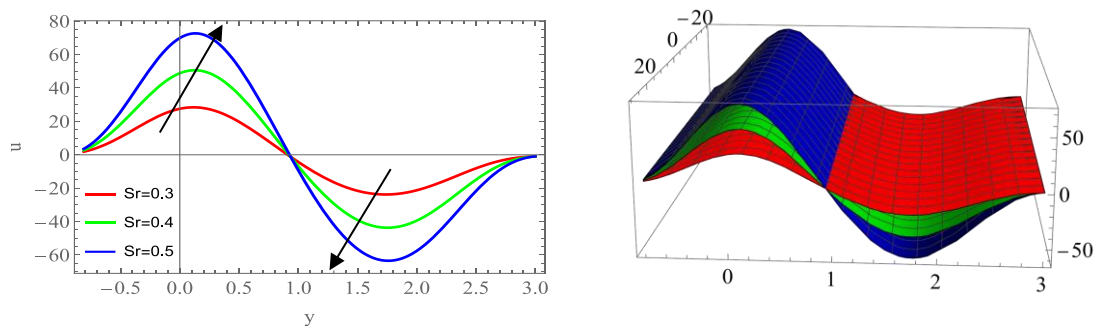


Figure 14. Variation of velocity u for different values of Sr when $Gr=3$, $Pr=0.02$, $M=0.2$, $\epsilon=0.02$, $\delta=0.001$, $k=0.02$, $Nt=0.05$, $U_{hs}=2$, $Kr=0.3$, $Q=2$, $Sc=3$, $h=1.2$, $Gc=0.04$, $\gamma=0.002$, $d=2$, and $x=1$

4. CONCLUSION

This study developed a mathematical model that could be used to study the joint influences of magnetic fields, heat transfer, and concentration gradients on SWCNT-based nanofluid flow in a ciliated microchannel. Numerical results are provided to show that the governing parameters play a significant role in the pressure distribution, velocity profiles, and temperature fields, which indicates a strong interdependence between the hydrodynamic, thermal, and mass-transfer mechanisms.

The model combines magnetohydrodynamic effects, thermophysical transport, and cilia-driven flow into a single model relevant to solar thermal energy systems. The results suggest that indicates proper control of magnetic intensity, thermal gradients, and nanoparticle transport can improve heat removal efficiency and enhance thermal regulation in solar collectors and PV/T systems.

While the present study is theoretical, the framework developed can be taken as the reference for testing experiments and implementation in actual solar-energy work. Future studies can be directed toward

integrating the proposed model with IoT technologies, embedded control systems, and smart monitoring platforms so that the regulation of nanofluid flow and thermal performance can be done in real time. Such an integration would be a step toward intelligent, energy-efficient, and sustainable solar thermal management systems.

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This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Ali Khalefa Hagi	✓	✓	✓	✓	✓	✓		✓	✓	✓			✓	
Liqaa Zeki Hummady		✓				✓		✓	✓	✓	✓	✓		

- C : Conceptualization
- M : Methodology
- So : Software
- Va : Validation
- Fo : Formal analysis
- I : Investigation
- R : Resources
- D : Data Curation
- O : Writing - Original Draft
- E : Writing - Review & Editing
- Vi : Visualization
- Su : Supervision
- P : Project administration
- Fu : Funding acquisition

CONFLICT OF INTEREST STATEMENT

The authors declare no conflict of interest

DATA AVAILABILITY

No datasets were generated or analyzed during the current study. The work is based on mathematical modeling and theoretical analysis.

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BIOGRAPHIES OF AUTHORS



Ali Khalefa Hagi    graduated from the Department of Mathematics, College of Science, University of Baghdad, Iraq, in 2005. He received his master's degree from the University of Baghdad in 2022 and is currently a Ph.D. student at the same university. He is a faculty member at Wasit University and currently teaches partial differential equations at the College of Education for Pure Sciences at Wasit University, Iraq. He has supervised numerous undergraduate research projects. He has co-authored two research papers in fluid mechanics. His research interests include fluid mechanics and nanofluids. He has also participated in several workshops and seminars. He has received multiple certificates of appreciation and letters of acknowledgment in recognition of his outstanding research and teaching performance. He can be contacted at email: ali.ali2303@sc.uobaghdad.edu.iq or alihagi@uowasit.edu.iq.



Asst. Prof. Dr. Liqaa Zeki Hummady    is a faculty member in the Department of Mathematics, College of Science, University of Baghdad, Iraq. She received her Ph.D. in applied mathematics and has extensive experience in teaching and research in fluid mechanics, heat and mass transfer, porous media, non-Newtonian fluids, mathematical modeling, and numerical analysis. Her research interests include computational mathematics, transport phenomena, continuum mechanics, stability analysis, and multiphysics systems. She has published numerous research papers in international journals and has supervised several postgraduate students. She actively participates in scientific conferences, academic committees, and research collaborations. Her current work focuses on advanced mathematical models involving magnetohydrodynamics, porous media flow, thermal transport, and biomedical applications. She is committed to promoting scientific research and higher education through teaching, supervision, and academic service. She can be contacted at email: liqaa.hummady@sc.uobaghdad.edu.iq.